Algebra I
Back Paper

Instructions. All questions carry ten marks.

1. Classify all groups of order 4.
2. Let G be a finite group. Prove that the number of group homomorphisms from the additive group of intergers $\mathbf{Z}$ to G equals the order of G.
3. Let p be a prime and n be a natural number. Prove that any group of order p^n has non-trivial center.
4. Define group action. Prove that none of the following can be a class equation of a group of order 10 :

   a. $10 = 1 + 2 + 3 + 4$
   b. $10 = 5 + 5$
   c. $10 = 1 + 1 + 1 + 2 + 5$
   d. $10 = 1 + 1 + 2 + 2 + 2 + 2$

5. State the three Sylow theorems. Prove that any group of order 15 must be cyclic.
6. Let p be a prime and G, H be two groups having n_p and m_p number of p-Sylow groups respectively. Prove that the number of p-Sylow subgroups of $G \times H$ is $n_p m_p$.